

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

<input type="text"/>									
----------------------	----------------------	----------------------	----------------------	----------------------	----------------------	----------------------	----------------------	----------------------	----------------------

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

BSA 1024 – STATISTICS
(All sections / Groups)

7 MARCH 2019
2.30 p.m – 4.30 p.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **ELEVEN (11)** printed pages with:
Section A: Ten (10) multiple choice questions (20%)
Section B: Three (3) structured questions (80%)
2. Answer **all** questions.
3. Answer **Section A** in the multiple-choice answer sheet provided and **Section B** in the answer booklet provided.
4. Formula and Statistical tables are attached at the end of the question paper.
5. Students are allowed to use non-programmable scientific calculators with no restrictions.

SECTION A: MULTIPLE CHOICE QUESTIONS (20 MARKS)

There are TEN (10) questions in this section. Answer ALL questions on the multiple-choice answer sheet.

1. Measurement for weight, height and length are classified as
 - A. measuring variables
 - B. continuous variables
 - C. qualitative variables
 - D. discrete variables

2. Which of the following is not a measure of variation?
 - A. mean
 - B. interquartile range
 - C. standard deviation
 - D. variance

3. If the standard deviation of a sample of 50 observations equals 6. The variance of the sample equals
 - A. 25
 - B. 12
 - C. 6
 - D. 36

4	7	5	12	12
5	6	8	5	6

Table 1

4. Refer to Table 1, calculate the median of the data.
 - A. 8.5
 - B. 5
 - C. 6
 - D. 10

5. Refer to Table 1, the variance is
 - A. 8.69
 - B. 7.22
 - C. 2.95
 - D. 6.51

6. If you flipped a coin, what is the probability of getting a head?
 - A. $\frac{1}{2}$
 - B. $\frac{1}{4}$
 - C. $\frac{1}{2}$
 - D. $\frac{3}{4}$

Continued...

7. Which of the following is not an example of a discrete probability distribution?
 - A. The sale price of a house
 - B. The number of bedrooms in a house
 - C. The number of bathrooms in a house
 - D. The number of swimming pool in a house
8. If $X \sim N(16, 49)$, then mean is
 - A. 49
 - B. 16
 - C. 7
 - D. 4
9. Confidence interval become narrow by increasing the
 - A. confidence interval
 - B. population size
 - C. degree of freedom
 - D. sample size
10. In hypothesis testing, a Type 2 error occurs when
 - A. The null hypothesis is not rejected when the null hypothesis is true.
 - B. The null hypothesis is rejected when the null hypothesis is true.
 - C. The null hypothesis is not rejected when the alternative hypothesis is true.
 - D. The null hypothesis is rejected when the alternative hypothesis is true.

Continued...

SECTION B: STRUCTURED QUESTIONS (80 MARKS)

There are THREE (3) questions in this section. Candidates MUST answer ALL THREE questions.

Question 1 (25 Marks)

a) The total amount of strawberry ice cream sold per day for one week is given as below.

Day	Relative Frequency
1	10
2	15
3	13
4	14
5	16
6	12
7	20
Total	100

- i) Prepare the probability distribution table for the total amount of strawberry ice cream sold per day. (3.5 marks)
- ii) Calculate the expected value and standard deviation. (5.5 marks)

- b) A survey showed that 30% of kids received their presents on their birthday. If 10 kids are selected randomly, find the probability that:
 - i) only 5 kids received their presents on their birthday. (3 marks)
 - ii) less than 3 of them did not receive their presents on their birthday. (3 marks)

- c) The number of cracks in a glass has a Poisson distribution with a mean of 1.50.
 - i) What is the probability that a glass has no cracks? (3 marks)
 - ii) Find the standard deviation of this distribution. (2 marks)

- d) The amount of time taken (in minutes) by Fahmi to reach campus from his house is normally distributed with a mean of 20 minutes and a standard deviation of 5 minutes. What is the probability that he will take more than 25 minutes to reach his campus? (5 marks)

Continued...

Question 2 (25 Marks)

a) In a recent study of 50 students in a secondary school, the mean number of hours per week that students surfing internet was 18.5 hours. The past testing show that the population standard deviation is 4.2 hours. Assume the population has a normal distribution.

- Construct a 90% confidence interval for the population mean, μ . (5 marks)
- Construct a 95% confidence interval for the population mean, μ . (5 marks)
- At 95% confidence level, how large a sample should be selected if they want the estimate to be within 2 hours of the population mean? (5 marks)

b) A manager of a bakery factory claims that the thickness of a cheesecakes produced is less than 7.95 inches. A quality control specialist who regularly checks this claim took a random sample of 10 cheesecakes and measured their thickness. The result obtained as follow:

7.91	7.89	7.98	7.91	7.90
7.90	7.96	7.92	7.93	7.95

Assume that the population standard deviation is 0.036. Using a 2.5% significance level, would you conclude that the manager's claim is true? (10 marks)

Question 3 (30 Marks)

a) An independent random samples of 17 matriculation students and 13 A-level students attending the same university and yield the following data on grade point averages (GPAs):

Matriculation			A-Level		
3.04	2.92	2.86	2.56	3.47	2.65
1.71	3.6	3.49	2.77	3.26	3
3.3	2.28	3.11	2.7	3.2	3.39
2.88	2.82	2.13	3	3.19	2.58
2.11	3.03	3.27	2.98		
2.6	3.13				

At the 5% significance level, do the data provide sufficient evidence to conclude that the mean GPAs of matriculation students and A-Level students at the university differ? (14 marks)

Continued...

b) In the manufacturing of a plastic material, it is believed that the cooling time has an influence on the impact strength. Therefore, a study is carried out in which plastic material impact strength is determined for 4 different cooling times. The results of this experiment and the excel regression analysis summary output are given below:

Cooling time in seconds (x)	15	25	35	40
Impact strength in kJ/m ² (y)	48	27	48	16

Regression Statistics	
Multiple R	0.5303
R Square	0.2812
Adjusted R Square	-0.0781
Standard Error	16.5565
Observations	4
Standard	
Coefficients	Error
Intercept	56.67797
Cooling times (X)	-0.762711
	0.862191
	-0.88462
	0.469683

i) Write down the regression equation. (2 marks)

ii) Estimate the plastic material impact strength if the cooling times is 30 minutes. (2 marks)

iii) Estimate the plastic material impact strength if the cooling times is 60 minutes. (2 marks)

c) A basket of goods consumed by a typical household consists of sugar, butter and eggs. The quantities consumed of each of these four goods in the base year and current year are given below along with the prices of these four goods in both the base year and the current year. Compute the Laspeyres Price Index (LPI), the Paasche Price Index (PPI) and the Fisher's Price index for 2018 by using 2017 as the base period.

Goods	2017		2018	
	P	Q	P	Q
A	6	24	5	31
B	4	7	3	11
C	3	9	2	9
D	11	3	2	6

(10 marks)

End of Page

STATISTICAL FORMULAE

A. DESCRIPTIVE STATISTICS

$$\text{Mean } (\bar{x}) = \frac{\sum_{i=1}^n X_i}{n}$$

$$\text{Standard Deviation } (s) = \sqrt{\frac{\sum_{i=1}^n X_i^2}{n-1} - \frac{(\sum_{i=1}^n X_i)^2}{n(n-1)}}$$

$$\text{Coefficient of Variation } (CV) = \frac{\sigma}{\bar{X}} \times 100$$

$$\text{Pearson's Coefficient of Skewness } (S_k) = \frac{3(\bar{X} - \text{Median})}{s}$$

B. PROBABILITY

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A) \times P(B) \quad \text{if } A \text{ and } B \text{ are independent}$$

$$P(A | B) = P(A \text{ and } B) / P(B)$$

Poisson Probability Distribution

$$\text{If } X \text{ follows a Poisson Distribution, } P(\lambda) \text{ where } P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\text{then the mean} = E(X) = \lambda \text{ and variance} = VAR(X) = \lambda$$

Binomial Probability Distribution

$$\text{If } X \text{ follows a Binomial Distribution } B(n, p) \text{ where } P(X=x) = {}^n C_x p^x q^{n-x}$$

$$\text{then the mean} = E(X) = np \text{ and variance} = VAR(X) = npq \text{ where } q = 1-p$$

Normal Distribution

$$\text{If } X \text{ follows a Normal distribution, } N(\mu, \sigma^2) \text{ where } E(X) = \mu \text{ and } VAR(X) = \sigma^2$$

$$\text{then } Z = \frac{X - \mu}{\sigma}$$

C. EXPECTATION AND VARIANCE OPERATORS

$$E(X) = \sum [X \bullet P(X)]$$

$$VAR(X) = E(X^2) - [E(X)]^2 \quad \text{where } E(X^2) = \sum [X^2 \bullet P(X)]$$

$$\text{If } E(X) = \mu \text{ then } E(cX) = c\mu, \quad E(X_1 + X_2) = E(X_1) + E(X_2)$$

$$\text{If } VAR(X) = \sigma^2 \text{ then } VAR(cX) = c^2 \sigma^2,$$

$$VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2 COV(X_1, X_2)$$

$$\text{where } COV(X_1, X_2) = E(X_1 X_2) - [E(X_1) E(X_2)]$$

D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

$(100 - \alpha)\%$ Confidence Interval for Population Mean (σ Known) =

$$\mu = \bar{X} \pm Z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right)$$

$(100 - \alpha)\%$ Confidence Interval for Population Mean (σ Unknown) =

$$\mu = \bar{X} \pm t_{\alpha/2, n-1} \left(\frac{s}{\sqrt{n}} \right)$$

$(100 - \alpha)\%$ Confidence Interval for Population Proportion = $\hat{p} \pm Z_{\alpha/2} \sigma_p$

$$\text{Where } \sigma_p = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\text{Sample Size Determination for Population Mean} = n \geq \left[\frac{(Z_{\alpha/2})\sigma}{E} \right]^2$$

$$\text{Sample Size Determination for Population Proportion} = n \geq \frac{(Z_{\alpha/2})^2 \hat{p}(1-\hat{p})}{E^2}$$

Where E = Limit of Error in Estimation

E. HYPOTHESIS TESTING

One Sample Mean Test

Standard Deviation (σ) Known

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Standard Deviation (σ) Not Known

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

One Sample Proportion Test

$$z = \frac{\hat{p} - p}{\sigma_p} \quad \text{where } \sigma_p = \sqrt{\frac{p(1-p)}{n}}$$

Two Sample Mean Test

Standard Deviation (σ) Known

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Standard Deviation (σ) Not Known

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$\text{where } S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}$$

Two Sample Proportion Test

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \quad \text{where } p = \frac{X_1 + X_2}{n_1 + n_2}$$

where X_1 and X_2 are the number of successes from each population

F. REGRESSION ANALYSIS

Simple Linear Regression

Population Model: $Y = \beta_0 + \beta_1 X_1 + \varepsilon$

Sample Model: $\hat{y} = b_0 + b_1 x_1 + e$

Correlation Coefficient

$$r = \frac{\sum XY - \left[\frac{\sum X \sum Y}{n} \right]}{\sqrt{\left[\sum X^2 - \left(\frac{\sum X}{n} \right)^2 \right] \left[\sum Y^2 - \left(\frac{\sum Y}{n} \right)^2 \right]}} = \frac{COV(X, Y)}{\sigma_x \sigma_y}$$

ANOVA Table for Regression

Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	1	SSR	MSR = SSR/1
Error/Residual	$n - 2$	SSE	MSE = SSE/(n - 2)
Total	$n - 1$	SST	

Test Statistic for Significance of the Predictor Variable

$$t_i = \frac{b_i}{S_{b_i}} \text{ and the critical value} = \pm t_{\alpha/2, (n-p-1)}$$

Where p = number of predictor

G. INDEX NUMBERS

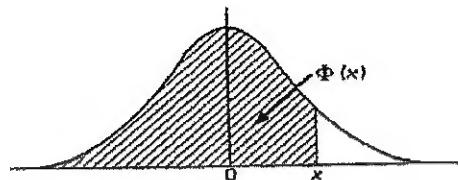
Simple Price Index $P = \frac{p_t}{p_0} \times 100$	Laspeyres Quantity Index $P = \frac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$
Aggregate Price Index $P = \frac{\sum p_t}{\sum p_0} \times 100$	Paasche Quantity Index $P = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$
Laspeyres Price Index $P = \frac{\sum p_t q_0}{\sum p_0 q_0} \times 100$	Fisher's Ideal Price Index $\sqrt{(Laspeyres Price Index)(Paasche Price Index)}$
Paasche Price Index $P = \frac{\sum p_t q_t}{\sum p_0 q_t} \times 100$	Value Index $V = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$

STATISTICAL TABLE

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$. $\Phi(x)$ is

the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x . When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$								
0.00	0.5000	0.40	0.6554	0.80	0.7881	1.20	0.8849	1.60	0.9452
0.01	0.5040	0.41	0.6591	0.81	0.7910	1.21	0.8869	1.61	0.9463
0.02	0.5080	0.42	0.6628	0.82	0.7939	1.22	0.8888	1.62	0.9474
0.03	0.5120	0.43	0.6664	0.83	0.7967	1.23	0.8907	1.63	0.9484
0.04	0.5160	0.44	0.6700	0.84	0.7995	1.24	0.8925	1.64	0.9495
0.05	0.5199	0.45	0.6736	0.85	0.8023	1.25	0.8944	1.65	0.9505
0.06	0.5239	0.46	0.6772	0.86	0.8051	1.26	0.8962	1.66	0.9515
0.07	0.5279	0.47	0.6808	0.87	0.8078	1.27	0.8980	1.67	0.9525
0.08	0.5319	0.48	0.6844	0.88	0.8106	1.28	0.8997	1.68	0.9535
0.09	0.5359	0.49	0.6879	0.89	0.8133	1.29	0.9015	1.69	0.9545
0.10	0.5398	0.50	0.6915	0.90	0.8159	1.30	0.9032	1.70	0.9554
0.11	0.5438	0.51	0.6950	0.91	0.8186	1.31	0.9049	1.71	0.9564
0.12	0.5478	0.52	0.6985	0.92	0.8212	1.32	0.9066	1.72	0.9573
0.13	0.5517	0.53	0.7019	0.93	0.8238	1.33	0.9082	1.73	0.9582
0.14	0.5557	0.54	0.7054	0.94	0.8264	1.34	0.9099	1.74	0.9591
0.15	0.5596	0.55	0.7088	0.95	0.8289	1.35	0.9115	1.75	0.9599
0.16	0.5636	0.56	0.7123	0.96	0.8315	1.36	0.9131	1.76	0.9608
0.17	0.5675	0.57	0.7157	0.97	0.8340	1.37	0.9147	1.77	0.9616
0.18	0.5714	0.58	0.7190	0.98	0.8365	1.38	0.9162	1.78	0.9625
0.19	0.5753	0.59	0.7224	0.99	0.8389	1.39	0.9177	1.79	0.9633
0.20	0.5793	0.60	0.7257	1.00	0.8413	1.40	0.9192	1.80	0.9641
0.21	0.5832	0.61	0.7291	1.01	0.8438	1.41	0.9207	1.81	0.9649
0.22	0.5871	0.62	0.7324	1.02	0.8461	1.42	0.9222	1.82	0.9656
0.23	0.5910	0.63	0.7357	1.03	0.8485	1.43	0.9236	1.83	0.9664
0.24	0.5948	0.64	0.7389	1.04	0.8508	1.44	0.9251	1.84	0.9671
0.25	0.5987	0.65	0.7422	1.05	0.8531	1.45	0.9265	1.85	0.9678
0.26	0.6026	0.66	0.7454	1.06	0.8554	1.46	0.9279	1.86	0.9686
0.27	0.6064	0.67	0.7486	1.07	0.8577	1.47	0.9292	1.87	0.9693
0.28	0.6103	0.68	0.7517	1.08	0.8599	1.48	0.9306	1.88	0.9699
0.29	0.6141	0.69	0.7549	1.09	0.8621	1.49	0.9319	1.89	0.9706
0.30	0.6179	0.70	0.7580	1.10	0.8643	1.50	0.9332	1.90	0.9713
0.31	0.6217	0.71	0.7611	1.11	0.8665	1.51	0.9345	1.91	0.9719
0.32	0.6255	0.72	0.7642	1.12	0.8686	1.52	0.9357	1.92	0.9726
0.33	0.6293	0.73	0.7673	1.13	0.8708	1.53	0.9370	1.93	0.9732
0.34	0.6331	0.74	0.7704	1.14	0.8729	1.54	0.9382	1.94	0.9738
0.35	0.6368	0.75	0.7734	1.15	0.8749	1.55	0.9394	1.95	0.9744
0.36	0.6406	0.76	0.7764	1.16	0.8770	1.56	0.9406	1.96	0.9750
0.37	0.6443	0.77	0.7794	1.17	0.8790	1.57	0.9418	1.97	0.9756
0.38	0.6480	0.78	0.7823	1.18	0.8810	1.58	0.9429	1.98	0.9761
0.39	0.6517	0.79	0.7852	1.19	0.8830	1.59	0.9441	1.99	0.9767
0.40	0.6554	0.80	0.7881	1.20	0.8849	1.60	0.9452	2.00	0.9772
								2.40	0.99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$										
2.40	0.99180	2.55	0.99461	2.70	0.99653	2.85	0.99781	3.00	0.99865	3.15	0.99918
2.41	0.99202	2.56	0.99477	2.71	0.99664	2.86	0.99788	3.01	0.99869	3.16	0.99921
2.42	0.99224	2.57	0.99492	2.72	0.99674	2.87	0.99795	3.02	0.99874	3.17	0.99924
2.43	0.99245	2.58	0.99506	2.73	0.99683	2.88	0.99801	3.03	0.99878	3.18	0.99926
2.44	0.99266	2.59	0.99520	2.74	0.99693	2.89	0.99807	3.04	0.99882	3.19	0.99929
2.45	0.99286	2.60	0.99534	2.75	0.99702	2.90	0.99813	3.05	0.99886	3.20	0.99931
2.46	0.99305	2.61	0.99547	2.76	0.99711	2.91	0.99819	3.06	0.99889	3.21	0.99934
2.47	0.99324	2.62	0.99560	2.77	0.99720	2.92	0.99825	3.07	0.99893	3.22	0.99936
2.48	0.99343	2.63	0.99573	2.78	0.99728	2.93	0.99831	3.08	0.99896	3.23	0.99938
2.49	0.99361	2.64	0.99585	2.79	0.99736	2.94	0.99836	3.09	0.99900	3.24	0.99940
2.50	0.99379	2.65	0.99598	2.80	0.99744	2.95	0.99841	3.10	0.99903	3.25	0.99942
2.51	0.99396	2.66	0.99609	2.81	0.99752	2.96	0.99846	3.11	0.99906	3.26	0.99944
2.52	0.99413	2.67	0.99621	2.82	0.99760	2.97	0.99851	3.12	0.99910	3.27	0.99946
2.53	0.99430	2.68	0.99632	2.83	0.99767	2.98	0.99856	3.13	0.99913	3.28	0.99948
2.54	0.99446	2.69	0.99643	2.84	0.99774	2.99	0.99861	3.14	0.99916	3.29	0.99950
2.55	0.99461	2.70	0.99653	2.85	0.99781	3.00	0.99865	3.15	0.99918	3.30	0.99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3.075	3.263	0.9994	3.731	0.99990	3.916	0.99995
3.105	3.320	0.9995	3.759	0.99991	3.976	0.99996
3.138	3.389	0.9996	3.791	0.99992	4.055	0.99997
3.174	3.450	0.9997	3.826	0.99993	4.173	0.99998
3.215	3.615	0.9998	3.867	0.99994	4.477	1.00000

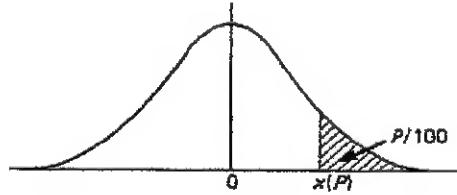
When $x > 3.3$ the formula $x - \Phi(x) = \frac{e^{-\frac{x^2}{2}}}{x\sqrt{2\pi}} \left[1 - \frac{1}{x^2} + \frac{3}{x^4} - \frac{15}{x^6} + \frac{105}{x^8} \right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}} \int_{x(P)}^{\infty} e^{-\frac{t^2}{2}} dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geq x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geq x(P)$ is $2P/100$.



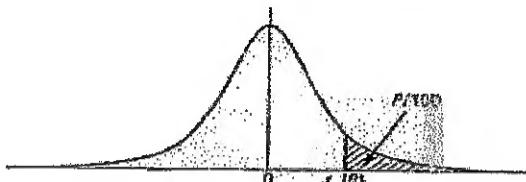
P	$x(P)$	P	$x(P)$								
50	0.0000	5.0	1.6449	3.0	1.8808	2.0	2.0537	1.0	2.3263	0.10	3.0902
45	0.1257	4.8	1.6646	2.9	1.8957	1.9	2.0749	0.9	2.3656	0.09	3.1214
40	0.2533	4.6	1.6849	2.8	1.9110	1.8	2.0969	0.8	2.4089	0.08	3.1559
35	0.3853	4.4	1.7060	2.7	1.9268	1.7	2.1201	0.7	2.4573	0.07	3.1947
30	0.5244	4.2	1.7279	2.6	1.9431	1.6	2.1444	0.6	2.5121	0.06	3.2389
25	0.6745	4.0	1.7507	2.5	1.9600	1.5	2.1701	0.5	2.5758	0.05	3.2905
20	0.8416	3.8	1.7744	2.4	1.9774	1.4	2.1973	0.4	2.6521	0.04	3.7190
15	1.0364	3.6	1.7991	2.3	1.9954	1.3	2.2262	0.3	2.7478	0.035	3.8906
10	1.2816	3.4	1.8250	2.2	2.0141	1.2	2.2571	0.2	2.8782	0.02	4.2649
5	1.6449	3.2	1.8522	2.1	2.0335	1.1	2.2904	0.1	3.0902	0.005	4.4172

TABLE 10. PERCENTAGE POINTS OF THE t -DISTRIBUTION

This table gives percentage points $t_r(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\pi}} \frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})} \int_{t_r(P)}^{\infty} \frac{dt}{(t + t^2/v)^{\frac{v+1}{2}}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2 -distribution with v degrees of freedom respectively; then $t = X_1/\sqrt{X_2/v}$ has Student's t -distribution with v degrees of freedom, and the probability that $t > t_r(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_r(P)$, and the probability that $|t| > s_r(P)$ is $2P/100$.



The limiting distribution of t as v tends to infinity is the normal distribution with zero mean and unit variance. When v is large interpolation in v should be harmonic.

P	40	30	25	20	15	10	5	2.5	1	0.5	0.2	0.05
$v = 1$	0.3249	0.7265	1.0000	1.3764	1.963	3.078	6.314	12.71	31.83	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.0607	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.182	4.543	5.841	10.31	12.92
4	0.2707	0.5686	0.7407	0.9410	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.2672	0.5594	0.7267	0.9295	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.2648	0.5534	0.7176	0.9057	1.134	1.440	1.943	2.447	3.143	3.707	5.203	5.959
7	0.2632	0.5491	0.7112	0.8960	1.119	1.415	1.895	2.365	2.998	3.499	4.781	5.408
8	0.2619	0.5459	0.7064	0.8889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.2610	0.5435	0.7027	0.8834	1.100	1.383	1.833	2.262	2.821	3.250	4.291	4.781
10	0.2602	0.5415	0.6998	0.8791	1.093	1.373	1.812	2.228	2.764	3.169	4.144	4.587
11	0.2596	0.5399	0.6974	0.8755	1.088	1.363	1.796	2.201	2.718	3.106	4.021	4.437
12	0.2590	0.5386	0.6955	0.8726	1.083	1.356	1.782	2.179	2.682	3.055	3.930	4.328
13	0.2586	0.5375	0.6938	0.8702	1.079	1.350	1.771	2.160	2.650	3.032	3.851	4.221
14	0.2582	0.5366	0.6926	0.8681	1.076	1.345	1.761	2.145	2.624	2.977	3.781	4.140
15	0.2579	0.5357	0.6912	0.8662	1.074	1.341	1.753	2.131	2.602	2.947	3.731	4.073
16	0.2576	0.5350	0.6901	0.8647	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.2573	0.5344	0.6892	0.8633	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.2571	0.5338	0.6884	0.8620	1.067	1.330	1.734	2.101	2.552	2.878	3.611	3.922
19	0.2569	0.5333	0.6876	0.8610	1.066	1.328	1.729	2.093	2.539	2.861	3.575	3.883
20	0.2567	0.5329	0.6870	0.8600	1.064	1.325	1.725	2.086	2.528	2.845	3.532	3.850
21	0.2565	0.5325	0.6864	0.8591	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.2564	0.5321	0.6858	0.8583	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.2563	0.5317	0.6853	0.8575	1.060	1.319	1.714	2.069	2.500	2.807	3.483	3.768
24	0.2562	0.5314	0.6848	0.8569	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.2561	0.5312	0.6844	0.8562	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.2560	0.5309	0.6840	0.8557	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.2559	0.5306	0.6837	0.8551	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.2558	0.5304	0.6834	0.8546	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.2557	0.5302	0.6830	0.8542	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.2556	0.5300	0.6828	0.8538	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.2555	0.5297	0.6823	0.8530	1.054	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.2553	0.5294	0.6818	0.8523	1.052	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.2552	0.5291	0.6814	0.8517	1.052	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.2551	0.5288	0.6810	0.8512	1.051	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.2550	0.5286	0.6807	0.8507	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.2547	0.5278	0.6794	0.8489	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.2545	0.5272	0.6786	0.8477	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.2539	0.5258	0.6765	0.8446	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
200	0.2533	0.5244	0.6745	0.8416	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291

